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# ADVANCED GCE
# MATHEMATICS (MEI) 4763
Mechanics 3

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**
- 8 page Answer Booklet
- Graph paper
- MEI Examination Formulae and Tables (MF2)

**Other Materials Required:**
None

**Wednesday 21 January 2009**
**Afternoon**

**Duration:** 1 hour 30 minutes

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**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **8** pages. Any blank pages are indicated.

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**1** **(i)** Write down the dimensions of force and density (which is mass per unit volume). **[2]**

The viscosity, $\eta$, of a fluid is defined by the equation

$$F = \frac{\eta A(v_2 - v_1)}{d}$$

where $F$ is the force acting over an area $A$, and $v_1$, $v_2$ are the velocities at two points a distance $d$ apart in the fluid.

**(ii)** Find the dimensions of viscosity. **[3]**

**(iii)** When a sphere of radius $a$ and density $\rho$ falls through a fluid with viscosity $\eta$, it reaches a terminal velocity $v$ given by $v = \frac{2a^2\rho g}{9\eta}$. Show that this formula is dimensionally consistent. **[3]**

The Reynolds number, $R$, for the flow of fluid round an obstruction of width $w$ is a dimensionless quantity given by

$$R = \rho w^{\alpha} v^{\beta} \eta^{\gamma}$$

where $v$ is the velocity of the flow, $\rho$ is the density of the fluid and $\eta$ its viscosity.

**(iv)** Find the values of $\alpha$, $\beta$ and $\gamma$. **[5]**

A designer is investigating the flow of air round an aircraft of width 25 moving with velocity 150, at a height where the air has density 0.4 and viscosity $1.6 \times 10^{-5}$ (all in SI units). A scale model of the aircraft, with width 5, is used in a wind tunnel at ground level, where the air has density 1.3 and viscosity $1.8 \times 10^{-5}$. The Reynolds number for the model must be the same as that for the full-sized aircraft.

**(v)** Find the velocity of flow required in the wind tunnel. **[3]**

**2** **(a)** Fig. 2 shows a light inextensible string of length 3.3 m passing through a small **smooth** ring R of mass 0.27 kg. The ends of the string are attached to fixed points A and B, where A is vertically above B. The ring R is moving with constant speed in a horizontal circle of radius 1.2 m, AR = 2.0 m and BR = 1.3 m.

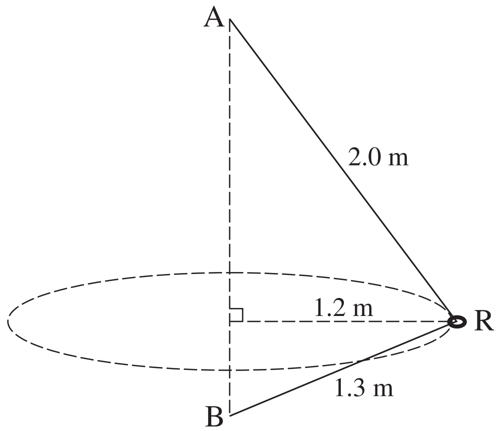


**Fig. 2**

**(i)** Show that the tension in the string is 6.37 N. **[5]**

**(ii)** Find the speed of R. **[4]**

**(b)** One end of a light inextensible string of length 1.25 m is attached to a fixed point O. The other end is attached to a particle P of mass 0.2 kg. The particle P is moving in a vertical circle with centre O and radius 1.25 m, and when P is at the highest point of the circle there is no tension in the string.

**(i)** Show that when P is at the highest point its speed is $3.5\ \mathrm{m\,s^{-1}}$. **[2]**

For the instant when the string OP makes an angle of $60^\circ$ with the upward vertical, find

**(ii)** the radial and tangential components of the acceleration of P, **[6]**

**(iii)** the tension in the string. **[2]**

**3** An elastic rope has natural length 25 m and modulus of elasticity 980 N. One end of the rope is attached to a fixed point O, and a rock of mass 5 kg is attached to the other end; the rock is always vertically below O.

**(i)** Find the extension of the rope when the rock is hanging in equilibrium. **[2]**

When the rock is moving with the rope stretched, its displacement is $x$ metres below the equilibrium position at time $t$ seconds.

**(ii)** Show that $\frac{d^2x}{dt^2} = -7.84x$. **[4]**

The rock is released from a position where the rope is slack, and when the rope just becomes taut the speed of the rock is $8.4 \text{ m s}^{-1}$.

**(iii)** Find the distance below the equilibrium position at which the rock first comes instantaneously to rest. **[4]**

**(iv)** Find the maximum speed of the rock. **[2]**

**(v)** Find the time between the rope becoming taut and the rock first coming to rest. **[4]**

**(vi)** State three modelling assumptions you have made in answering this question. **[3]**

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**4** **(a)** The region bounded by the $x$-axis and the semicircle $y = \sqrt{a^2 - x^2}$ for $-a \leqslant x \leqslant a$ is occupied by a uniform lamina with area $\frac{1}{2}\pi a^2$. Show by integration that the $y$-coordinate of the centre of mass of this lamina is $\dfrac{4a}{3\pi}$. **[4]**

**(b)** A uniform solid cone is formed by rotating the region between the $x$-axis and the line $y = mx$, for $0 \leqslant x \leqslant h$, through $2\pi$ radians about the $x$-axis.

**(i)** Show that the $x$-coordinate of the centre of mass of this cone is $\frac{3}{4}h$. **[6]**
[You may use the formula $\frac{1}{3}\pi r^2 h$ for the volume of a cone.]

From such a uniform solid cone with radius 0.7 m and height 2.4 m, a cone of material is removed. The cone removed has radius 0.4 m and height 1.1 m; the centre of its base coincides with the centre of the base of the original cone, and its axis of symmetry is also the axis of symmetry of the original cone. Fig. 4 shows the resulting object; the vertex of the original cone is V, and A is a point on the circumference of its base.

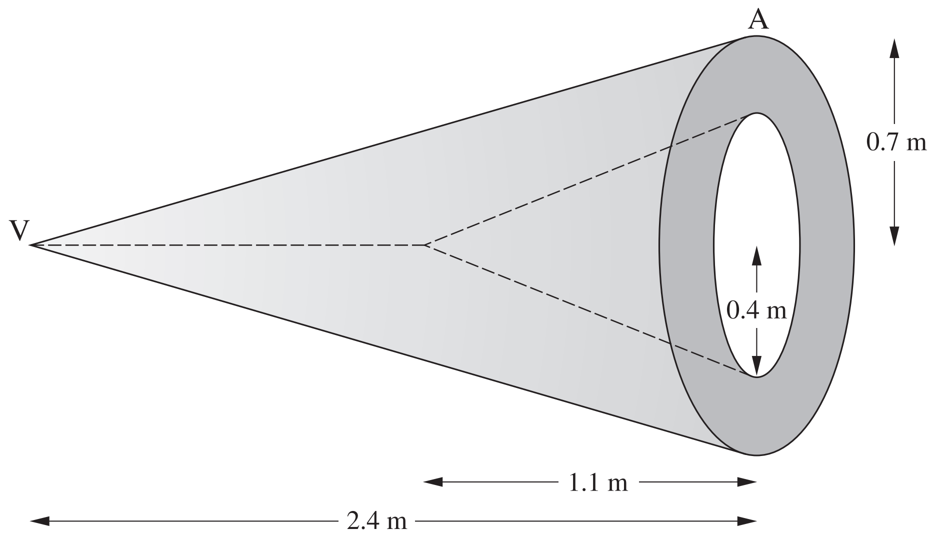


**Fig. 4**

**(ii)** Find the distance of the centre of mass of this object from V. **[5]**

This object is suspended by a string attached to a point Q on the line VA, and hangs in equilibrium with VA horizontal.

**(iii)** Find the distance VQ. **[3]**

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# 4763 Mechanics 3

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|---|---|---|---|
| **1 (i)** | [ Force ] $= \mathrm{M\,L\,T^{-2}}$<br>[ Density ] $= \mathrm{M\,L^{-3}}$ | B1<br>B1<br>**2** | |
| **(ii)** | $[\eta] = \dfrac{[F][d]}{[A][v_2 - v_1]} = \dfrac{(\mathrm{M\,L\,T^{-2}})(\mathrm{L})}{(\mathrm{L^2})(\mathrm{L\,T^{-1}})}$<br>$= \mathrm{M\,L^{-1}\,T^{-1}}$ | B1<br>M1<br>A1<br>**3** | for $[A] = \mathrm{L}^2$ and $[v] = \mathrm{L\,T^{-1}}$<br>Obtaining the dimensions of $\eta$ |
| **(iii)** | $\left[\dfrac{2a^2\rho g}{9\eta}\right] = \dfrac{\mathrm{L^2(M\,L^{-3})(L\,T^{-2})}}{\mathrm{M\,L^{-1}\,T^{-1}}} = \mathrm{L\,T^{-1}}$<br>which is same as the dimensions of $v$ | B1<br>M1<br>E1<br>**3** | For $[g] = \mathrm{L\,T^{-2}}$<br>Simplifying dimensions of RHS<br>Correctly shown |
| **(iv)** | $(\mathrm{M\,L^{-3}})\mathrm{L}^{\alpha}(\mathrm{L\,T^{-1}})^{\beta}(\mathrm{M\,L^{-1}\,T^{-1}})^{\gamma}$ is dimensionless<br>$\gamma = -1$<br>$-\beta - \gamma = 0$<br>$-3 + \alpha + \beta - \gamma = 0$<br>$\alpha = 1, \quad \beta = 1$ | B1 cao<br>M1<br>M1A1<br>A1 cao<br>**5** | |
| **(v)** | $R = \dfrac{\rho w v}{\eta} = \dfrac{0.4 \times 25 \times 150}{1.6 \times 10^{-5}} \quad (= 9.375 \times 10^{7})$<br>$= \dfrac{1.3 \times 5v}{1.8 \times 10^{-5}}$<br>Required velocity is $260\ \mathrm{m\,s^{-1}}$ | M1<br>A1<br>A1 cao<br>**3** | Evaluating $R$<br>Equation for $v$ |

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| **2 (a)(i)** | $T\cos\alpha = T\cos\beta + 0.27\times 9.8$<br>$\sin\alpha = \frac{1.2}{2.0} = \frac{3}{5}$, $\cos\alpha = \frac{4}{5}$ $(\alpha = 36.87°)$<br>$\sin\beta = \frac{1.2}{1.3} = \frac{12}{13}$, $\cos\beta = \frac{5}{13}$ $(\beta = 67.38°)$<br>$\frac{27}{65}T = 2.646$<br>Tension is 6.37 N | M1<br>A1<br>B1<br>M1<br>E1<br>**5** | Resolving vertically (weight and at least one resolved tension)<br>Allow $T_1$ and $T_2$<br>For $\cos\alpha$ and $\cos\beta$ [ or $\alpha$ and $\beta$ ]<br>Obtaining numerical equation for $T$ e.g. $T(\cos 36.9 - \cos 67.4) = 0.27\times 9.8$<br>*(Condone 6.36 to 6.38)* |
| **(ii)** | $T\sin\alpha + T\sin\beta = 0.27\times\frac{v^2}{1.2}$<br>$6.37\times\frac{3}{5} + 6.37\times\frac{12}{13} = 0.27\times\frac{v^2}{1.2}$<br>$v^2 = 43.12$<br>Speed is $6.57\ \text{m s}^{-1}$ | M1<br>A1<br>M1<br>A1<br>**4** | Using $v^2/1.2$<br>Allow $T_1$ and $T_2$<br>Obtaining numerical equation for $v^2$ |
| **(b)(i)** | $0.2\times 9.8 = 0.2\times\frac{u^2}{1.25}$<br>$u^2 = 9.8\times 1.25 = 12.25$<br>Speed is $3.5\ \text{m s}^{-1}$ | M1<br>E1<br>**2** | Using acceleration $u^2/1.25$ |
| **(ii)** | $\frac{1}{2}m(v^2 - 3.5^2) = mg(1.25 - 1.25\cos 60)$<br>$v^2 = 24.5$<br>Radial component is $\frac{24.5}{1.25}$<br>$= 19.6\ \text{m s}^{-2}$<br>Tangential component is $g\sin 60$<br>$= 8.49\ \text{m s}^{-2}$ | M1<br>A1<br>M1<br>A1<br>M1<br>A1<br>**6** | Using conservation of energy<br>With numerical value obtained by using energy<br>(M0 if mass, or another term, included)<br>For sight of $(m)g\sin 60°$ with no other terms |
| **(iii)** | $T + 0.2\times 9.8\cos 60 = 0.2\times 19.6$<br>Tension is 2.94 N | M1<br>A1 cao<br>**2** | Radial equation (3 terms)<br>*This M1 can be awarded in (ii)* |

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| **3 (i)** | $\frac{980}{25}y = 5\times 9.8$<br>Extension is 1.25 m | M1<br>A1<br>**2** | Using $\frac{\lambda y}{l_0}$ (Allow M1 for $980y = mg$ ) |
| **(ii)** | $T = \frac{980}{25}(1.25+x)$<br>$5\times 9.8 - 39.2(1.25+x) = 5\frac{d^2x}{dt^2}$<br>$-39.2x = 5\frac{d^2x}{dt^2}$<br>$\frac{d^2x}{dt^2} = -7.84x$ | B1 (ft)<br>M1<br>F1<br>E1<br>**4** | *(ft) indicates ft from previous parts as for A marks*<br>Equation of motion with three terms<br>Must have $\ddot{x}$ In terms of $x$ only |
| **(iii)** | $8.4^2 = 7.84(A^2 - 1.25^2)$<br>Amplitude is 3.25 m | M2<br>A1<br>A1<br>**4** | Using $v^2 = \omega^2(A^2 - x^2)$ |
| | OR M2<br>$\frac{980}{2\times 25}y^2 = 5\times 9.8y + \frac{1}{2}\times 5\times 8.4^2$ A1<br>$y = 4.5$<br>Amplitude is $4.5 - 1.25 = 3.25$ m A1 | | Equation involving EE, PE and KE |
| | OR $x = A\sin 2.8t + B\cos 2.8t$<br>$x = -1.25,\ v = 8.4$ when $t = 0$<br>M2<br>$\Rightarrow A = 3,\ B = -1.25$ A1<br>Amplitude is $\sqrt{A^2+B^2} = 3.25$ A1 | | Obtaining $A$ and $B$<br>Both correct |
| **(iv)** | Maximum speed is $A\omega = 3.25\times 2.8$<br>$= 9.1\ \mathrm{m\,s^{-1}}$ | M1<br>A1<br>**2** | or equation involving EE, PE and KE<br>ft only if answer is greater than 8.4 |
| **(v)** | $x = 3.25\cos 2.8t$<br>$-1.25 = 3.25\cos 2.8t$<br>Time is 0.702 s | B1 (ft)<br>M1<br>M1<br>A1 cao<br>**4** | or $x = 3.25\sin 2.8t$<br>or $v = 9.1\cos 2.8t$ or $v = 9.1\sin 2.8t$<br>or $x = 3.25\sin(2.8t+\varepsilon)$ etc<br>or $x = \pm 3\sin 2.8t \pm 1.25\cos 2.8t$<br>Obtaining equation for $t$ or $\varepsilon$ by setting $x = (\pm)1.25$ or $v = (\pm)8.4$ or solving<br>$\pm 3\sin 2.8t \pm 1.25\cos 2.8t = 3.25$<br>Strategy for finding the required time<br>e.g. $\frac{1}{2.8}\sin^{-1}\frac{1.25}{3.25} + \frac{1}{4}\times\frac{2\pi}{2.8}$<br>$2.8t - 0.3948 = \frac{1}{2}\pi$ or<br>$2.8t - 1.966 = 0$ |

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| **(vi)** | e.g. Rope is light<br>Rock is a particle<br>No air resistance / friction / external forces<br>Rope obeys Hooke's law / Perfectly elastic / Within elastic limit / No energy loss in rope | B1B1B1<br>**3** | Three modelling assumptions |

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| **4 (a)** | $\int \frac{1}{2}y^2\,\mathrm{d}x = \int_{-a}^{a} \frac{1}{2}(a^2-x^2)\,\mathrm{d}x$ | M1 | For integral of $(a^2-x^2)$ |
| | $= \left[\ \frac{1}{2}(a^2x-\frac{1}{3}x^3)\ \right]_{-a}^{a}$ | | |
| | $=\frac{2}{3}a^3$ | A1 | |
| | $\bar{y} = \dfrac{\frac{2}{3}a^3}{\frac{1}{2}\pi a^2}$ | M1 | *Dependent on previous M1* |
| | $= \dfrac{4a}{3\pi}$ | E1<br>**4** | |
| **(b)(i)** | | | *$\pi$ may be omitted throughout* |
| | $V = \int \pi y^2\,\mathrm{d}x = \int_0^h \pi(mx)^2\,\mathrm{d}x$ | M1 | For integral of $x^2$ or use of $V=\frac{1}{3}\pi r^2 h$ and $r = mh$ |
| | $= \left[\ \frac{1}{3}\pi m^2x^3\ \right]_0^h = \frac{1}{3}\pi m^2h^3$ | A1 | |
| | $\int \pi xy^2\,\mathrm{d}x = \int_0^h \pi x(mx)^2\,\mathrm{d}x$ | M1 | For integral of $x^3$ |
| | $= \left[\ \frac{1}{4}\pi m^2x^4\ \right]_0^h = \frac{1}{4}\pi m^2h^4$ | A1 | |
| | $\bar{x} = \dfrac{\frac{1}{4}\pi m^2h^4}{\frac{1}{3}\pi m^2h^3}$ | M1 | *Dependent on M1 for integral of $x^3$* |
| | $=\frac{3}{4}h$ | E1<br>**6** | |
| **(ii)** | $m_1 = \frac{1}{3}\pi\times0.7^2\times2.4\rho = \frac{1}{3}\pi\rho\times1.176$ | | |
| | $\mathrm{VG}_1 = 1.8$ | | |
| | $m_2 = \frac{1}{3}\pi\times0.4^2\times1.1\rho = \frac{1}{3}\pi\rho\times0.176$ | B1 | For $m_1$ and $m_2$ (or volumes) |
| | $\mathrm{VG}_2 = 1.3+\frac{3}{4}\times1.1 = 2.125$ | B1 | or $\frac{1}{4}\times1.1$ from base |
| | $(m_1-m_2)(\mathrm{VG})+m_2(\mathrm{VG}_2) = m_1(\mathrm{VG}_1)$ | M1 | Attempt formula for composite body |
| | $(\mathrm{VG})+0.176\times2.125 = 1.176\times1.8$ | F1 | |
| | Distance (VG) is 1.74 m | A1<br>**5** | |
| **(iii)** | VQG is a right-angle | M1 | |
| | $\mathrm{VQ} = \mathrm{VG}\cos\theta$ where $\tan\theta = \dfrac{0.7}{2.4}$ $(\theta = 16.26°)$ | M1 | |
| | $\mathrm{VQ} = 1.7428\times\dfrac{24}{25}$ | | |
| | $= 1.67$ m | A1<br>**3** | ft is $\mathrm{VG}\times0.96$ |

# 4763 Mechanics 3

### General Comments

This paper was found to be somewhat harder than the January 2008 paper. Nevertheless, there were many excellent scripts, demonstrating a sound knowledge of all the topics being examined, and about 30% of the candidates scored 60 marks or more (out of 72). Circular motion and simple harmonic motion continue to cause problems for a considerable number of candidates, and it was common for a candidate to lose most of the marks in question 2 or question 3, or both.

### Comments on Individual Questions

1 Dimensional analysis
This question was answered very well indeed, with about 70% of the candidates scoring full marks.

(i) Almost every candidate gave the dimensions of force and density correctly.

(ii) Almost all candidates knew how to find the dimensions of viscosity, although there were a few slips here.

(iii) This was also well done. Some candidates did not complete their argument convincingly, for example by failing to mention the dimensions of velocity, or by leaving in the numerical factor 2/9. Those who had incorrect dimensions for viscosity usually asserted that the dimensions of their right-hand-side simplified to $LT^{-1}$ when they clearly did not; but a few candidates used this part to correct a previous error.

(iv) The method for finding the indices was well understood, although there were quite a number of slips (usually sign errors) made when forming and solving the simultaneous equations.

(v) The use of the Reynolds number to calculate the velocity was very well understood, and almost every candidate who had the right formula for R was able to complete this correctly.

2 Circular motion
About one third of the candidates scored full marks on this question, and the average mark was 14 out of 19.

(a)(i) The tension was very often found correctly, although there were many misunderstandings in this part. Some assumed that the two sections of the string were perpendicular, and some had only one term involving the tension.

(ii) Those who had answered the first part correctly almost always found the speed correctly in this part. Similarly, any misunderstandings in the first part were usually repeated here.

(b)(i) Almost all candidates could find the speed at the highest point correctly.

(ii) Many candidates began this part by writing down the radial equation of motion (which is not required until part (iii)), but most then realised that they needed to apply conservation of energy. The energy equation often contained errors, such as incorrect signs and sine and cosine muddles, in the potential energy term. Many candidates knew the formula $g\sin\theta$ for the tangential component of acceleration and could apply it efficiently, although there were quite a few who tried to do something with the formula $r\dot{\omega}$ obtained from the formula book.

(iii) Most candidates understood how to find the tension in the string.

3 Simple harmonic motion

Very few candidates scored full marks on this question, and the average mark was about 10 out of 19.

(i) Almost every candidate found the extension correctly with the rock in equilibrium.

(ii) Many candidates omitted this part, and some just quoted a formula such as $\omega^2 = \lambda/lm$ . Candidates were expected to apply Newton's second law, showing the weight and the tension as separate terms; even when this was done, sign errors occurred quite frequently.

(iii) The simplest approach was to use $v^2=\omega^2(A^2-x^2)$ to find the amplitude, but very few candidates did this. Most used energy, often successfully, but there were many errors such as using x in the gravitational or elastic terms where it should be $(1.25+x)$; sometimes one form of energy was completely omitted.

(iv) Most candidates understood how to find the maximum speed, usually from the formula $A\omega$ (although some used energy again). Those with an incorrect amplitude often obtained an answer which was less than 8.4, but did not appear to realise that this must be wrong.

(v) This part was found very difficult. The time can be found by solving the equation $3.25\cos 2.8t=-1.25$, but most of the successful candidates used much more complicated strategies. The most common response was to give one quarter of the period.

(vi) The expected responses were: the rock is a particle; the rope is light; there is no air resistance; the rope obeys Hooke's law. However, few candidates could give more than two of these.

4 Centres of mass

About 15% of the candidates scored full marks on this question, and the average mark was 12 out of 18.

(a) Most candidates obtained the centre of mass of the lamina correctly.

(b)(i) Most candidates integrated correctly to obtain $(\frac{1}{3}\pi r^2 h)\bar{x} = \frac{1}{4}\pi m^2 h^4$ . However, many did not see that $r=mh$, and so were unable to complete the proof.

(ii) The method was well understood, but here a very common error was to take the distance from V of the centre of mass of the cone removed to be ¾×1.1 instead of 1.3+¾×1.1. There was also some confusion about which end of the cone the ¾*h* was measured from.

(iii) This part was reasonably well answered, although quite a few put the right-angle in the triangle VQG at G instead of Q.